

Earliness/Tardiness Production Problem using Semi-Infinite Optimization

Problema de Producción temprana/tardía usando Optimización Semi-infinita

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Abstract

Introduction— An Earliness/Tardiness Production Problem is considered, with the objective of delivering products of a company in an optimum time. Based on the problem settled in [1], which was solved with a genetic method, we develop this semi-infinite optimization problem (SIP) with the stochastic outer approximations method created by [2].

Objective— Obtain a better solution of the problem or at least get a similar solution is the goal of this work.

Methodology— With regard to the numerical solution a stochastic outer approximation algorithm was used, in addition numerical experiments were performed in MATLAB.

Results— By last, the algorithm developed will be applied in a real case of Colombian production Company, to give the optimum setting for production planning.

Conclusions— The solution obtained in this work has a bigger function value in contrast with the solutions obtained in [1], [3], but it satisfies all constrains.

Keywords— Earliness/Tardiness production; semi-infinite optimization; outer approximations methods; just-in-time; stochastic algorithms

Resumen

Introducción— Se considera el problema de producción temprana/tardía con el objetivo de distribuir los productos a los clientes en los tiempos óptimos. Basado en el problema descrito en [1], que fue resuelto por ellos con los algoritmos genéticos, el problema se presenta como uno de programación semi-infinita (SIP).

Objetivo— Obtener la mejor solución del problema de la producción temprana/tardía o como mínimo una similar a esta.

Metodología— El método estocástico de aproximaciones externas fue aplicado para obtener solución numérica del problema y los algoritmos fueron programados en MATLAB.

Resultados— Algoritmo propuesto se aplica para un caso real de producción en Colombia y brinda la solución óptima de la planeación de producción.

Conclusiones— La solución obtenida en este trabajo da el valor de la función objetivo ligeramente mayor en comparación con las soluciones obtenidas en [1] y [3], pero cumple todas las restricciones.

Palabras clave— Producción temprana/tardía; optimización semi-infinita; métodos de aproximaciones externas; justo-a-tiempo; algoritmos estocásticos

I. INTRODUCTION

In this article we are going to show one way to find a solution of a Semi-Infinite Optimization Problem (SIP), which is based on inventory management method called Just-in-Time (JIT). The Just-in-time production is a manufacturing process that prioritize arrival's time of materials, goods, and labor that are scheduled in the production process. JIT aligns raw-material orders from suppliers directly with production schedules, so that, materials are bought exactly in the time and amount needed, this reduces costs by minimizing warehouse needs. JIT was developed in Japan between 1960 and 1970, particularly at Toyota's company [4]-[6].

In a JIT system we seek the reduction of the in-process inventory level every second of production labor, which means an infinite number of limitations, that encourage us to introduce the concept of semi-infinite optimization problem [3]. A SIP is an optimization problem with a finite number of variables and an infinite number of constraints [3], [7]. A typical problem with a continua number of constraints is the semi-infinite programming problem (1):

$$\begin{aligned} \text{SIP: } \quad & f(x) \rightarrow \min_x \\ & \text{such that (s.t.) } g(x, t) \leq 0 \quad \forall t \in T^0 \\ & \quad \quad \quad x \in X^0 \end{aligned} \quad (1)$$

Where f and g are assumed to be continuously differentiable on a neighborhood of $X^0 \times T^0$, $X^0 \times \mathbb{R}^k$ and $T^0 \subset \mathbb{R}^k$ are convex and compact and $\mathbb{R}^k$ is the k -dimensional real space.

II. PROBLEM DESCRIPTION

Let us consider a company that should produce orders in a specific time interval $[0, T]$. This example was taken from [1]. For each order i with $1 \leq i \leq n$ and time $t \in [0, T]$, define: the decision variable x_i , the production cycle L_i , the due date d_i , the function of required resources $R_i(t, x_i)$ (3), the function that measures the amount of accessible resources $G(t)$ and we give to each order a weight α_i , which is set by its price.

Then we should solve the below SIP (2):

$$\begin{aligned} \min_{x_i} \quad & \sum_{i=1}^n \alpha_i (x_i - d_i)^2 \\ \text{s.t.} \quad & \sum_{i=1}^n R_i(t, x_i) \leq G(t) \\ & 0 \leq L_i \leq x_i \leq T \end{aligned} \quad (2)$$

Note that the problem (2) has n variables and the fact that belongs to $[0, T]$ guarantees the existence of infinite constrains.

In order to simplify the problem, we will assume that required resource function behaves as a normal distribution, then it should have the form:

$$R_i(t, x_i) = a_i \exp \left[\frac{-(t - x_i + b_i)^2}{c_i} \right] \quad (3)$$

For $1 \leq i \leq n$. When $x_i = L_i$ the function (3) should fulfill the condition (4):

$$\int_0^{L_i} R_i(t, L_i) dt = \delta P_i \quad (4)$$

With δ a relaxation coefficient between 0 and 1, and P_i the total amount of required resources for the i order. Then if we take $\delta = 0.97$ and for normal distribution properties, we have (5) from [8]:

$$\begin{aligned} a_i &= \frac{P_i}{\frac{L_i}{4}\sqrt{2\pi}} \\ b_i &= \frac{L_i}{2} \\ c_i &= \frac{L_i^2}{8} \end{aligned} \quad (5)$$

Also, let us must consider maintenance jobs in the production labor and that they spend some available resources, then suppose that there are maintenance jobs which take place in the time interval previously defined. Every maintenance job requires $Q_j(t)$ amount of resource in time t , with $1 \leq j \leq m$, we used an analog normal distribution to simulate Q_j along time. Then we end up with the total able resource function:

$$G(t) = g_0 \exp(t\beta) - \sum_{j=1}^m Q_j(t) \quad (6)$$

And (6) should satisfy

$$\begin{aligned} G(t) &\geq \max\{a_i | 1 \leq i \leq n\} \\ \int_0^T G(t) dt &\geq \sum_{i=1}^n P_i \end{aligned} \quad (7)$$

So that our SIP makes sense.

So, we search a vector of optimum time of deliveries, in such a way that the distance between each deliver and the due-date is smaller if those deliveries guarantee that the amount of available resources is bigger than the amount of needed resources.

In the next section we introduce the algorithm that we are going to use to solve the problem.

III. ALGORITHM

The stochastic outer approximation method was developed by [2], it is a fixed multi-start method that improves the random search method developed by [9] to solve this type of SIP with continua of constrains.

A. New Version of the Method

To use the method, we must define some specific sets. The set (8) of all feasible points linked with the set $[0, T]$ is:

$$X^0 = \{x_i \in [0, T] | g(x_i, t) \leq 0\} \quad (8)$$

The set of optimal points is (9):

$$X_{opt} = \{x_i \in X^0 | f(x_i) = \min_{x' \in X^0} f(x')\} \quad (9)$$

In the same way we can define X^n , $t \in T_n$, where $T_n \subset [0, T]$ and T_n is finite subset and X_{opt}^n is the subset of optimal points.

Quasi-Optimal Function. Define the function $\Theta: X^0 \times M_c([0, T]) \rightarrow \mathbb{R}$ (10):

$$\Theta(x_i, T) = \max \left\{ 0, \max_{t \in T} g(x_i, t), f(x_i) - \min_{y \in X} f(y) \right\} \quad (10)$$

Where $X \in M_c(X^0)$ and $M_c([0, T])$ is the set of all compact subsets of $[0, T]$.

We can check that if $x \in X_{\text{opt}}$ implies $\Theta(x, [0, T]) = 0$. We proceed to describe the new algorithm.

SMETH.ETPP illustrate the steps to find an optimal solution of the problem:

Step0:

Input:
 $x \in X^0$

Define $\hat{T} = \emptyset, i = 1, n = 1$.

Parameters $\gamma, \{\epsilon_n\}, \{\sigma_n\}$:

$\epsilon_n, \sigma_n > 0, n = 1, 2, \dots, \epsilon_n, \sigma_n \searrow 0$

Step1:

Choose a random value for t_i^n , include that value in $\hat{T}$, then, with that point, search t_i^{n*} with a descent method such that

$$g(x, t_i^{n*}) = \max_{t \in \hat{T}} g(x, t),$$

include t_i^{n*} in $\hat{T}$.

Step2:

Define $\theta_i = \Theta(x, \hat{T})$

Step3: (Stopping criteria)

IF $\theta_i \leq \gamma$

THEN define $i = i + 1$

IF $i > 500$

THEN the current x is consider a solution of the problem. Stop running.

ELSE go to *Step1*

Step4: Define $\overline{T}_n = \hat{T}$

$$\Theta_n = \theta_i,$$

$$S_n = i.$$

Step5:

Use relevant parameters and ignore the others, include relevant ones in

$$\Delta T_n \subset T_n$$

Step6:

Define

$$T_{n+1} = \Delta T_n \cup \bigcup_{\substack{\theta_j < \sigma_j \\ 1 \leq j \leq n-1}} \Delta T_j$$

Step7:

Find x^{n+1} such that satisfies:

$$\Theta(x^{n+1}, T_{n+1}) < \epsilon_{n+1}$$

Step8:

Do $n = n + 1$

Go to *Step1*.

It can be seen that SMETH.ETPP is also a Multi-start method. The main difference between this algorithm and the master method is the inclusion of a final step. The master method does not have a proper step where ending of the algorithm was contemplated. We add a restriction to the number of iterations in *Step 3* to finish properly the whole program.

IV. NUMERICAL RESULTS

Using the stochastic outer approximation method considering a SIP with time as the infinite set of constraints. The same problem described here was solved in [1], with a specially designed genetic algorithm with mutation along the negative gradient direction and in [3] implementing simulated annealing method combined with a heuristic and the steepest descent method.

A. Solution of Test Problem

Utilizing the documentation of MathWorks with some tools from MATLAB Optimization Toolbox, we proceed to resolve the described Earliness/Tardiness production problem, with the proposed algorithm.

The example deals with a construction company which has scheduled orders for different building constructions. The main resource constraint is measuring manpower. The maximum time considered is $T = 100$ weeks. The available manpower at the beginning of the operation process is $g_0 = 100$ and it increases at a rate of $\beta = 0.005$. There are two previously scheduled reparation jobs A and B. For job A, the manpower needed is kilo-hours, it is supposed to be ended at the 40th week and it takes weeks to finish it. For job B, the manpower used is kilo-hours, it is supposed to be finished at the 70th week and it takes 40 weeks to finish it.

The construction cycle, manpower requirement, contract price, and due-date of each order are shown in the next Table 1:

TABLE 1. TEST PROBLEM.

	Constructing	Resource	Contract	Desired	Completion
No.	cycle	requirement	price	due-date	time
1	20	400	10	25	20.00
2	20	900	18	30	24.91
3	30	800	12	35	78.86
4	40	800	15	40	56.91
5	25	1000	28	40	39.46
6	20	1200	20	40	86.77
7	50	2000	30	50	70.74
8	10	300	18	15	10.87
9	20	400	9	50	76.28
10	60	1500	30	60	91.07

Source: Authors.

B. Reduce and Put two graphs

Then, the equations for infinite constraints take the form (11) and (12):

$$R_i(t, x_i) = \frac{p_i}{\frac{L_i}{4}\sqrt{2\pi}} \exp \left[\frac{-(t - x_i - \frac{L_i}{2})^2}{\frac{L_i^2}{8}} \right] \quad (11)$$

For the resource requirement function in each order, and:

$$Q_j(t) = \frac{D_j}{\frac{W_j}{4}\sqrt{2\pi}} \exp \left[-\frac{(t - J_j - \frac{W_j}{2})^2}{\frac{W_j^2}{8}} \right] \quad (12)$$

For the resource requirement function in each maintenance job, where Q_1 and Q_2 are the functions related to the maintenance jobs A and B respectively. And as mentioned in the Problem Description is the total resource required in each job, W_j is the amount of time needed to finish each job and J_j is the specific dates to make the maintenance jobs, then in our example, we have:

$$D = [200 \ 300],$$

$$W = [20 \ 40],$$

$$J = [40 \ 70].$$

Finally define $g(x_i, t)$ as in (2). In the following figures Fig. 1 and Fig. 2, the resources function is showed both before and after the optimization method was implemented, respectively.

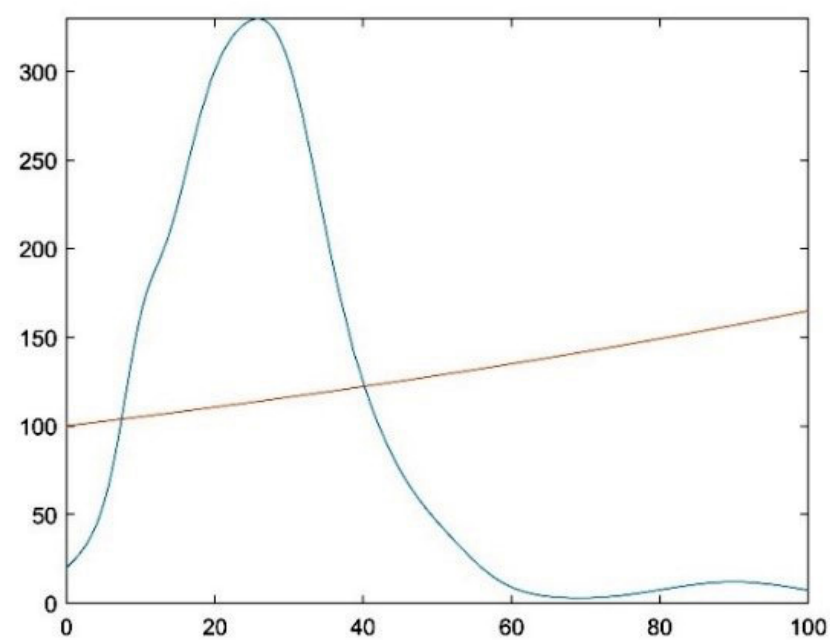


Fig. 1. Before optimization.
Source: Authors.

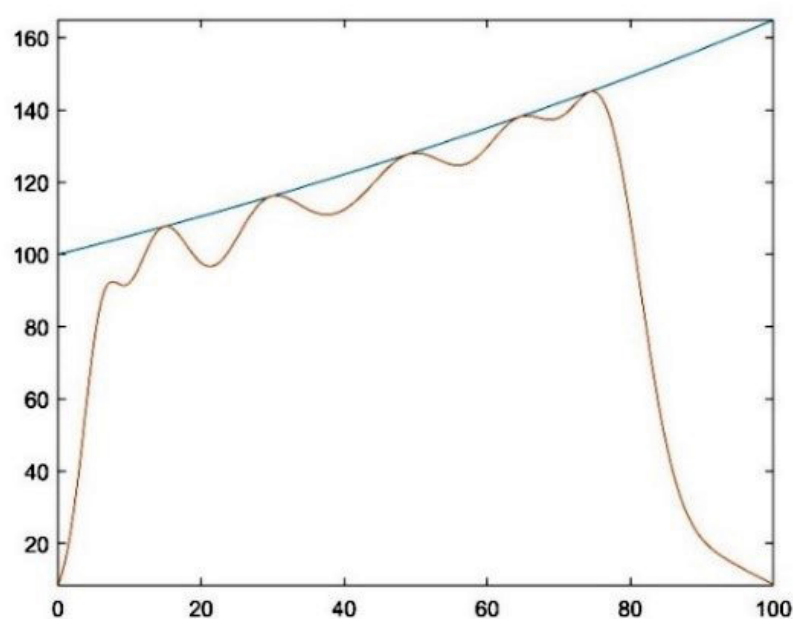


Fig. 2. After optimization process.
Source: Authors.

C. Contrasting with Other Methods

After compiling the program times we get the following solutions (Table 2).

TABLE 2. RESULTS OF TEST PROBLEM.

Run	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	fval
1	20.0	26.3447	62.6751	47.9448	47.5759	90.3885	88.7749	12.1021	36.0468	90.9367	1.3874×10 ⁵
2	34.3619	20.8391	38.5331	41.3357	69.5913	48.5448	98.1959	10.0	58.6149	10.0	1.4735×10 ⁵
3	20.0	24.9111	78.8632	56.9186	39.4681	86.7720	70.7481	10.8756	76.2842	91.0797	1.2027×10 ⁵
4	20.0	33.5344	30.0	80.4647	47.6352	88.9213	87.7073	12.3049	52.6507	83.1788	1.3380×10 ⁵
5	20.0	39.4704	88.6194	40.0	30.5299	51.1553	91.9886	11.6539	66.2885	98.3180	1.4089×10 ⁵
6	20.0	24.5612	91.6899	76.0440	55.1606	39.8767	91.1598	10.5010	31.1119	94.0920	1.5454×10 ⁵
7	20.0	41.3153	64.1375	85.8654	52.1792	30.4859	93.4855	11.4980	20.0	95.5472	1.5322×10 ⁵
8	20.8032	37.5206	30.0	40.0	65.8604	49.3244	95.2214	10.7018	77.8988	97.7424	1.3338×10 ⁵
9	20.0	31.6034	30.0	86.7238	60.3743	42.9003	95.1213	12.1698	51.5021	99.0656	1.5216×10 ⁵
10	20.0	31.5877	88.3384	40.0	61.9669	45.3155	92.5767	10.0	20.0921	96.4595	1.5127×10 ⁵

Source: Authors.

Here we can observe that solution in the 3th run correspond to the minimal value of the function f . This solution shows us the best time in which each order must be delivered. In our example this means, the time to finish each construction order. If we denote our solution as x_{EG} the objective function takes the value:

$$f(x_{ET}) = 1.2027 \times 10^5.$$

In [1] their optimization process reaches the next solution of the SIP:

$$x_g = [30.74 \ 20.76 \ 41.88 \ 40.79 \ 52.50 \ 86.77 \ 84.38 \ 10.00 \ 75.75 \ 87.28]$$

And the function takes the value:

$$f(x_g) = 1.1477 \times 10^5$$

Which means that $f(x_g) < f(x_{ET})$ but the solution x_g has active constrains in the interval (73, 76.3), then $g(x_g, t) > 0$ for $t \in (73, 76.3)$ this fact implies that is not an optimum solution for the problem.

In [3] their optimization process reach the next solution of the SIP:

$$x_g = [31.41 \ 20.56 \ 40.01 \ 40.86 \ 55.60 \ 10.03 \ 43.77 \ 83.30].$$

The function takes the value:

$$f(x_g) = 1.1296 \times 10^5$$

which means that $f(x_g) < f(x_{ET})$ but the solution x_g has active constrains in the intervals $(5.7, 8.5) \cup (11, 16.2) \cup (17, 24.7) \cup (39.7, 49.7) \cup (73.9, 76.3)$, then $(x_g, t) > 0$ this fact implies that is not a optimum solution for the problem.

D. Real Case

We take data from a Colombian Company that produces refrigerator parts. We applied the outer stochastic approximation method in this Company to optimize the number of workers assigned to different products. The interval of time considered is one month of production. This refrigerator manufacturer has monotonous assembly operations. Just-in-time production planning is ideal for factories which their process is repetitive and systematic [4], because of that we proceed to apply JIT production method in this company.

The data that we will consider is based on 16 products elaborated in the factory, in each one the cycle time it takes to be finished was measured. Data set obtained after measuring is the following (Table 3):

TABLE 3. DATA OF REAL CASE.

Code	Cycle (Hrs.)	Resources (Pers.)	Due-date (Hrs.)	Price (Cop)	(Hrs.)
30842092	51.72	60	72	22'503.600	156.18
30842101	81.66	60	120	38'301.900	151.51
30842102	124.66	60	144	53'120.100	200.04
30842104	141.66	60	168	64'420.000	201.72
30864015	74.5	160	360	19'999.080	465.59
30864030	77.17	160	96	22'362.480	124.84
30515111	25.27	80	360	8'224.000	302.87
30515112	80	80	120	28'632.000	98.84
30515110	2.13	40	240	512.900	721.13
30515103	1.63	40	120	1'852.600	721.13
30515104	1.63	40	120	1'065.600	721.13
30515102	1.63	40	240	1'385.900	721.13
30530107	516.81	80	696	184'817.598	517.68
30530106	209.72	80	360	80'156.840	335.59
30505096	353.16	80	480	129'165.300	354.04
30505122	23.80	110	480	101'584.600	312.72

Where Cycle means the time that takes in hours to produce each product. The resources column is the number of people required in each product. Due-date is measured in hours. The price is measured in Colombian pesos. We assumed a rate of resource's increasing of 0.001. As in the test example, for solving the SIP with JIT model, we will prioritize the different orders by their sell-value. We assume the same objective function and same required resource function as in test example.

Before applying the stochastic outer approximation method to solve the set SIP in this Company, we observe the below graphic of the resource's function among time (Fig. 3):

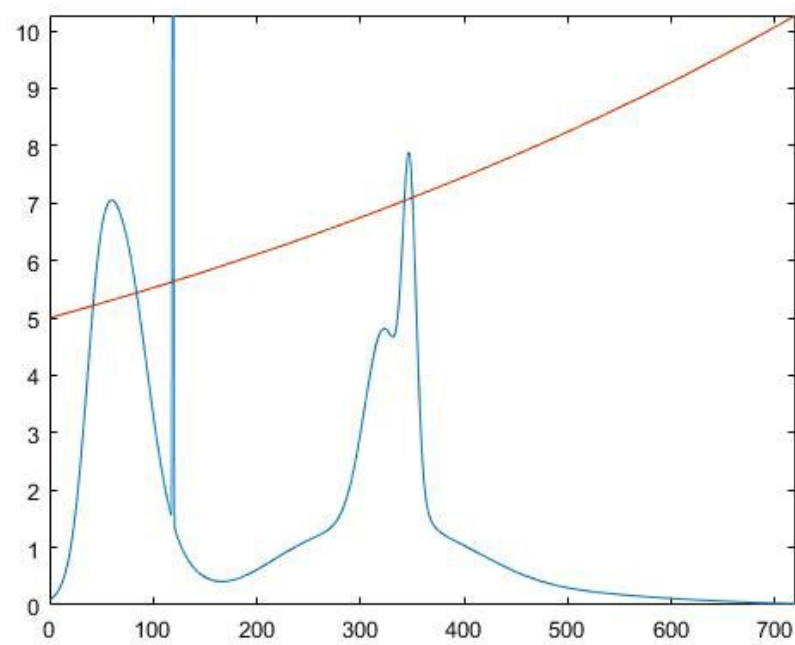


Fig. 3. Resource's function before optimization.

Source: Authors.

The Fig. 3 is a non-convex function and the pick showed around 100 hours means that the company need a big number of workers in that specific time, which implies that more resources are used in that time and there is a sudden increase in production costs. What we try to fix is that necessity, searching a better time for the company to deliver their products and maintain the cost of elaborate new artifacts steady among time.

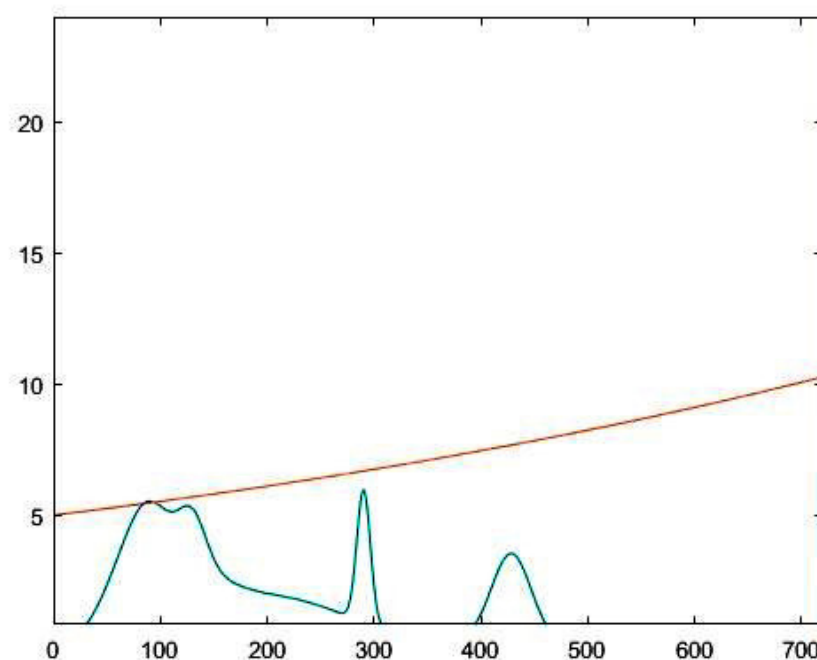


Fig. 4. Resource's function after optimization.
Source: Authors.

Fig. 4 represents the same resource's function after applying the stochastic outer approximation method. With this final solution we can guarantee that the amount of needed resources will be always less or equal to the amount of available resources. To accomplish this objective function is important for the company to have a short number of suppliers and guarantee that each of them shift the materials or resources required in exact time.

E. Adjustment of Parameters

To solve the SIP using SMETH.ETPP, we made some assumptions. For the required resource function, we used a function of normal distribution and a relaxation coefficient equal to 1 (13):

$$\int_0^{L_i} R_i(t, L_i) dt = \delta P_i = P_i \quad (13)$$

These types of functions are not likely to appear in real world factories. Other assumption made in the model is that there is a function that measures the amount of available resources in the company, and such function has an exponential form and without maintenance jobs pre-scheduled (14):

$$G(t) = g_0 \exp(t\beta) \quad (14)$$

These assumptions made the SIP easier to develop and solve.

As we can see in Fig. 3 there was a high pick in the resource function before implementing the optimization method, so the code used for this problem needed to be changed in the dropping of active constrains, because dropping constrains near to the pick, will ignore a large number of active constrains and will end up with a bad solution of the minimization problem. It was necessary to drop fewer active constrains and change the tolerance parameter while clustering, this change affected the number and duration of program's iterations compared to the iterations made in test problem.

IV. FINANCEMENT PROJECT

Project name: Optimización de producción temprana/tardanza mediante programación semi-infinita. Code: 38696. Universidad Nacional de Colombia. Sede Bogotá. Date: 24-08-2017 / 27-07-2019.

V. CONCLUSIONS

The solution obtained in this work has a bigger function value in contrast with the solutions obtained in [1] and [3] but it satisfies all constraints.

It is necessary to develop a faster way to drop active constraints in cases where the function of constraints has peaks or is not convex.

Just-in-time management plan is well posed on a company like the used in the real case because setting a steady number of orders is easy and suppliers will know the amount of resources required in advance.

While implementing the stochastic outer approximation method in a test example is quite direct and natural, applying the same method in a real-world problem becomes much harder. In real life is almost impossible to find smooth functions that tell us the number of available and required resources along time.

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